Kuwait University — Math 102 — August 16, 1997

Dept. of Math. and Comp. Sc. — Final Examination — Duration: 120 minutes

Calculators and mobile phones are not allowed.
Answer all of the following questions.

1. Let $f(x) = \ln(1 + 2^x)$. Show that f^{-1} exits and find $f^{-1}(x)$. Find also the domain and range of f^{-1}. (6 points)

2.(a) Find $\frac{dy}{dx}$, if $y = \frac{x^{\sin^{-1} x}}{e^{\sinh x}\, 3^x}$

(b) Find the limit, if it exits: $\lim_{x\to\infty} (1 - \frac{e}{x})^x$ (3+3 points)

3. Evaluate the following integrals

(a) $\int \frac{1}{2 - \sin x}\, dx$, (b) $\int e^x \tan^{-1} e^x dx$

(c) $\int \frac{x^2}{(x-1)^3} dx$.

(5+4+4 points)

4. Determine whether the following integral is convergent or divergent. Find its value, if convergent.

$$\int_0^{\frac{\pi}{2}} \frac{\tan x}{1 + \ln(\cos x)} dx.$$

(5 points)

5. Given are two curves by the polar equations $r_1 = 4$ and $r_2 = 2 + 2\cos\theta$.

(a) Sketch the graph of the curves and label the points on the axes $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

(b) Find the area of the region that is inside the graph of r_1 and outside the graph of r_2. (3+3 points)

6. Find the arc length of the curve defined parametrically by the equations $x(t) = t - \tanh t$, $y(t) = \operatorname{sech} t$ for $0 \le t \le 2$. (4 points)

7.(a) Given points $P(1, 1, 0)$, $Q(-1, 1, -2)$ and $R(0, 2, -1)$, find the component of $\overrightarrow{PQ}$ along $\overrightarrow{PR}$. (3 points)

(b) Show that the lines

(L_1): $x = 4 + 2t$ $y = 2 + 3t$ $z = 8 + 5t$

(L_2): $x = 5 + 3u$ $y = -2 - u$ $z = 5 + 2u$

intersect and find an equation of the plane they determine. (3+4 points)

Total 50 points.